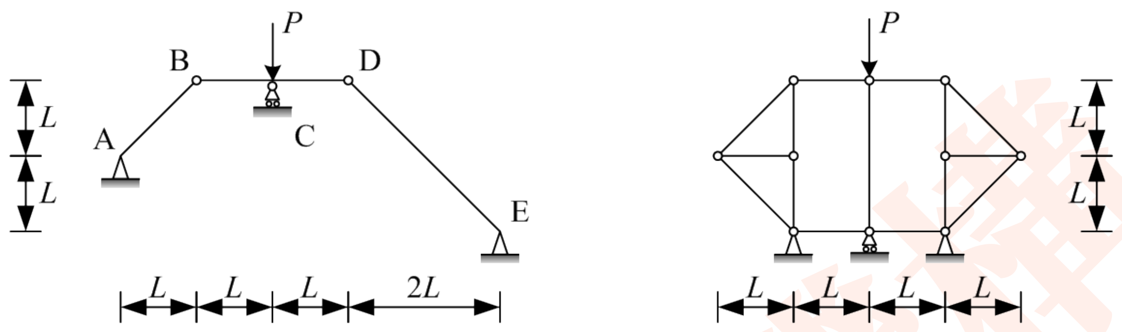
100 年鐵路人員特別考試高員三級結構學參考解答

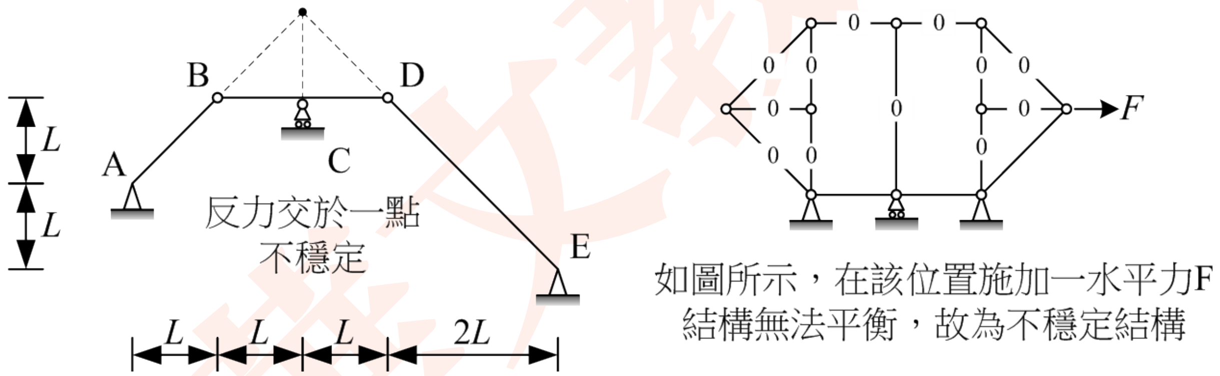
一、請判別圖一(a)及圖一(b)中之結構。有小圈圈處代表鉸接點，否則為剛接。它們是穩定結構嗎？請說明你的判斷邏輯，否則不予計分；如果是穩定結構，它的超靜定次數（static indeterminacy）為若干？請說明你的判斷邏輯，否則不予計分，並求出應有之反力。(20 分)



圖一(a)　　圖一(b)

(100 鐵路高員-結構學 #1)

【參考解答】



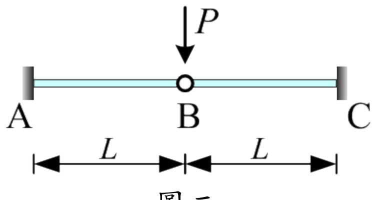
【說明】

(1)在圖一(a)結構中，當外力作用在 BCD 桿件時，二力桿 AB、DE 與支承 C 的反力會交交於一點，形成幾何不穩定結構系統。題目所給定之 P 恰好作用在反力交點上，剛好形成『不穩定平衡』。

(2)在圖一(b)結構中，利用『任意載重法』加載並消除零桿後，會發現節點無法滿足靜平衡方程式，故為一不穩定結構系統。題目所給定之 P 恰好可形成『不穩定平衡』

(3)一個穩定結構必須在任意載重下皆為穩定平衡，而不是在特定載重下維持穩定平衡。

二、設有一如圖二所示之複合梁 ABC，A、C 兩點為固定端，B 點為鉸接，梁之斷面彎曲剛度為 EI，B 點承受集中載重 P。試用共軛梁法（conjugated beam method）求 B 點之垂直變位 δ_B 及相對轉角 θ_B。（25 分）

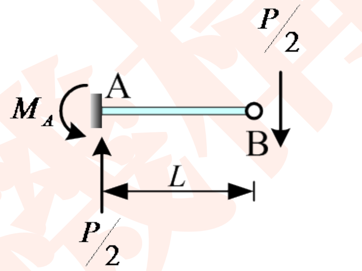


圖二

(100 鐵路高員-結構學 #2)

【參考解答】

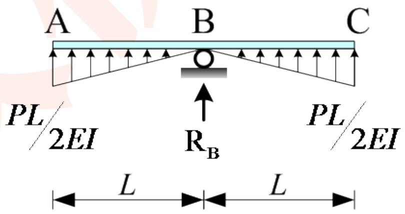
(1)結構內力分析



結構為對稱，A、B 兩固端支承反力為 $\frac{1}{2}P$

取 AB 自由體：$\Sigma_B = 0 \;\Rightarrow\; \frac{P}{2}\times L = M_A = \frac{PL}{2}$

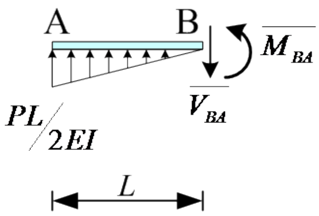
(2)繪製共軛梁圖，計算 $\overline{R_B}$



$$\Sigma F_y = 0 \;\Rightarrow\; 2\left(\frac{1}{2}\times\frac{PL}{2EI}\times L\right) = \overline{R_B}$$

$$\therefore\; \overline{R_B} = \frac{PL^2}{2EI}\;(\downarrow)$$

(3) 取共軛梁之 AB 段自由體計算 $\overline{V_{BA}}$，$\overline{M_{BA}}$



$$\Sigma F_y = 0 \;\Rightarrow\; \frac{1}{2}\times\frac{PL}{2EI}\times L = \overline{V_{BA}}$$

$$\therefore\; \overline{V_{BA}} = \frac{PL^2}{4EI}\;(\downarrow)$$

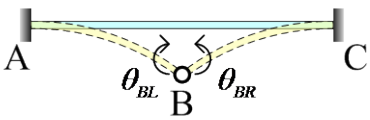
$$\Sigma M_B = 0 \;\Rightarrow\; \left(\frac{1}{2}\times\frac{PL}{2EI}\times L\right)\times\frac{2}{3}L = \overline{M_{BA}}$$

$$\therefore\; \overline{M_{BA}} = \frac{PL^3}{6EI}\;(\circlearrowleft)$$

(4)求解 δ_B

$$\delta_B = \overline{M_{BA}} = \frac{PL^3}{6EI}\;(\downarrow)$$

(5)求解 B 點相對轉角 θ_B

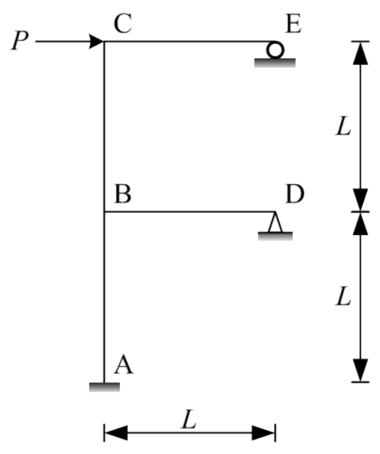


$$\theta_B = \overline{R_B} = \frac{PL^2}{2EI}\;(\curvearrowright\cdot\curvearrowleft)$$

$$\theta_{BL} = \frac{PL^2}{4EI}\;(\circlearrowright)$$

$$\theta_{BR} = \frac{PL^2}{4EI}\;(\circlearrowleft)$$

三、設有如圖三之剛架，各桿斷面彎曲剛度均為 EI。試用傾角變位法公式（slope-deflection equation）解此剛架，且繪此剛架之軸向力圖（N-dia.）、剪力圖（V-dia.）、彎矩圖（M-dia.）及彈性變形曲線。（30 分）



圖三

(100 鐵路高員-結構學 #3)

【參考解答】

(1)桿件勁度比

$$k_{AB}:k_{BD}:k_{BC}:k_{CE}=\frac{2EI}{L}:\frac{2EI}{L}:\frac{2EI}{L}:\frac{2EI}{L}=1:1:1:1$$

(2)固端彎矩

無

(3)自由度

$$\theta_B\ ，\ \theta_C\ ，\ R_{BC}=\frac{(\Delta_C)_h}{L}=R$$

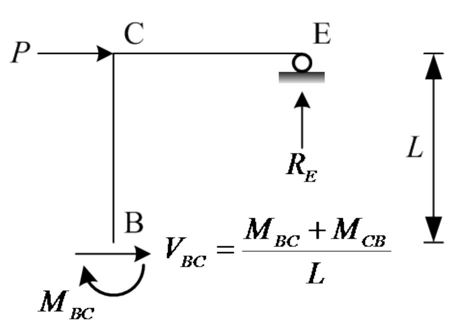
(4)傾角變位式

$$M_{AB}=1\cdot(\theta_B)$$
$$M_{BA}=1\cdot(2\theta_B)$$
$$M_{BD}=1\cdot(1.5\theta_B)$$
$$M_{BC}=1\cdot(2\theta_B+\theta_C-3R)$$
$$M_{CB}=1\cdot(\theta_B+2\theta_C-3R)$$
$$M_{CE}=1\cdot(1.5\theta_C)$$

(5)平衡方程式

$$\Sigma M_B=0 \Rightarrow M_{BA}+M_{BD}+M_{BC}=0$$
$$\Rightarrow 5.5\theta_B+\theta_C-3R=0 \cdots\cdots\cdots ①$$
$$\Sigma M_C=0 \Rightarrow M_{BC}+M_{CE}=0$$
$$\Rightarrow \theta_B+3.5\theta_C-3R=0 \cdots\cdots\cdots ②$$

取 BCE 自由體



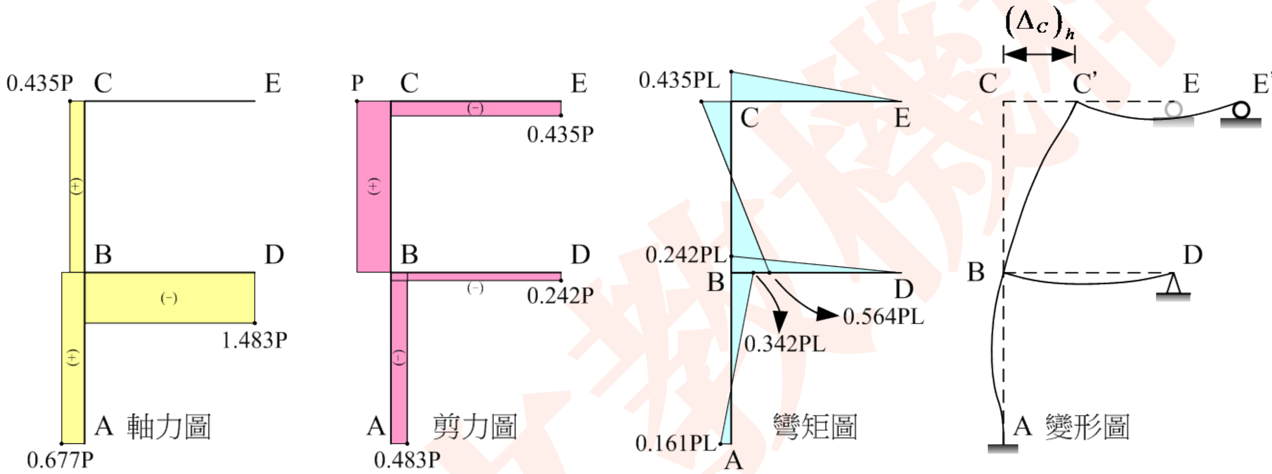
$$\Sigma F_x=0 \Rightarrow P+V_{BC}=0$$
$$\Rightarrow 3\theta_B+3\theta_C-6R=-PL \quad \cdots\cdots ③$$

聯立①②③可解得：$\theta_B = 0.161PL$，$\theta_C = 0.290PL$，$R = 0.392PL$

將θ_B、θ_C、R帶回傾角變位式，可得各桿端彎矩

$$\Rightarrow \begin{cases} M_{AB} = 0.161PL \\ M_{BA} = 0.322PL \\ M_{BD} = 0.242PL \\ M_{BC} = -0.564PL \\ M_{CB} = -0.435PL \\ M_{CE} = 0.435PL \end{cases}$$

(6)繪製結構軸力圖、剪力圖、彎矩圖及彈性變形曲線

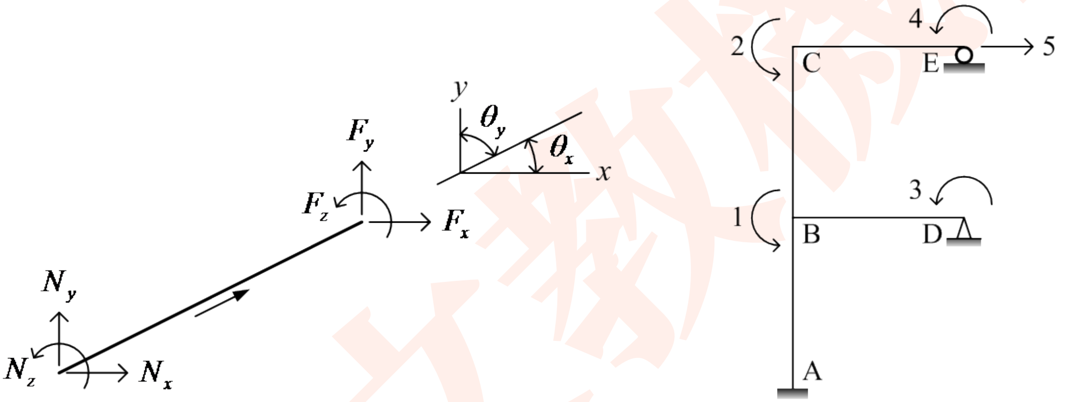


四、設有如圖三所示之剛架（第三題中之剛架），試求對應於圖四中所標示之五個自由度之勁度矩陣 $K_{5\times5}$。（提示：桿件勁度矩陣，k。）（25 分）

桿件勁度矩陣，k：

$$
k=\begin{array}{c}
\begin{array}{cccccc} N_x & N_y & N_z & F_x & F_y & F_z \end{array} \\
\left[\begin{array}{cccccc}
\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y & -\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\frac{6EI}{L^2}\lambda_y \\
\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & \frac{6EI}{L^2}\lambda_x \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{4EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} \\
-\left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & -\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y & \left(\frac{AE}{L}\lambda_x^2+\frac{12EI}{L^3}\lambda_y^2\right) & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \frac{6EI}{L^2}\lambda_y \\
-\left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & -\left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x & \left(\frac{AE}{L}-\frac{12EI}{L^3}\right)\lambda_x\lambda_y & \left(\frac{AE}{L}\lambda_y^2+\frac{12EI}{L^3}\lambda_x^2\right) & -\frac{6EI}{L^2}\lambda_x \\
-\frac{6EI}{L^2}\lambda_y & \frac{6EI}{L^2}\lambda_x & \frac{2EI}{L} & \frac{6EI}{L^2}\lambda_y & -\frac{6EI}{L^2}\lambda_x & \frac{4EI}{L}
\end{array}\right]
\end{array}
\begin{array}{c} \\ N_x \\ N_y \\ N_z \\ F_x \\ F_y \\ F_z \end{array}
$$

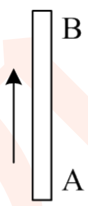
$\lambda_x=\cos\theta_x$ ， $\lambda_y=\cos\theta_y$



圖四

(100 鐵路高員-結構學 #4)

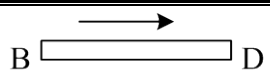
【參考解答】



$\theta_x=90^\circ\Rightarrow\lambda_x=0$

$\theta_y=0^\circ\Rightarrow\lambda_y=1$

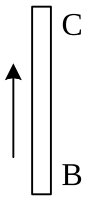
$$
\begin{array}{cc}
 & ① \\
 & F_Z \\
[k_{AB}] = & \left[\frac{4EI}{L}\right] \; F_Z \quad ①
\end{array}
$$



$\theta_x=0^\circ\Rightarrow\lambda_x=1$

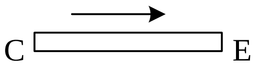
$\theta_y=90^\circ\Rightarrow\lambda_y=0$

$$
[k_{BD}]=\begin{array}{c}
\begin{array}{cc} ① & ③ \\ N_Z & F_Z \end{array} \\
\begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix}
\end{array}
\begin{array}{c} N_Z \quad ① \\ F_Z \quad ③ \end{array}
$$



$$\theta_x = 90^\circ \Rightarrow \lambda_x = 0$$
$$\theta_y = 0^\circ \Rightarrow \lambda_y = 1$$

$$[k_{BC}] = \begin{array}{c} \begin{array}{cc} ① & ② \\ N_Z & F_Z \end{array} \\ \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{array}{l} \\ \\ N_Z \quad ① \\ F_Z \quad ② \end{array}$$



$$\theta_x = 0^\circ \Rightarrow \lambda_x = 1$$
$$\theta_y = 90^\circ \Rightarrow \lambda_y = 0$$

$$[k_{CE}] = \begin{array}{c} \begin{array}{ccc} ② & ⑤ & ④ \\ N_Z & F_x & F_Z \end{array} \\ \begin{bmatrix} \frac{4EI}{L} & 0 & \frac{2EI}{L} \\ 0 & \frac{EA}{L} & 0 \\ \frac{2EI}{L} & 0 & \frac{4EI}{L} \end{bmatrix} \end{array} \begin{array}{l} \\ \\ N_Z \ ② \\ F_x \ ⑤ \\ F_Z \ ④ \end{array}$$

$$[K] = \frac{\mathrm{E}}{\mathrm{L}} \begin{bmatrix} 4I+4I+4I & 2I & 2I & 0 & 0 \\ 2I & 4I+4I & 0 & 2I & 0 \\ 2I & 0 & 4I & 0 & 0 \\ 0 & 2I & 0 & 4I & 0 \\ 0 & 0 & 0 & 0 & A \end{bmatrix}$$

$$= \frac{\mathrm{E}}{\mathrm{L}} \begin{bmatrix} 12I & 2I & 2I & 0 & 0 \\ 2I & 8I & 0 & 2I & 0 \\ 2I & 0 & 4I & 0 & 0 \\ 0 & 2I & 0 & 4I & 0 \\ 0 & 0 & 0 & 0 & A \end{bmatrix}$$