

一、【參考題解】

設泥沙密度為 ρ_s ，直徑為 d_s ，體積為 ∇_s ，泥沙顆粒投影面積為 A ，泥沙下降的終端速度為 V ，阻力係數為 C_D ，水密度為 ρ_w

由力平衡知：
$$W = F_b + F_D \quad (1)$$

其中泥沙重 $W = \gamma_s \nabla_s = \rho_s g \nabla_s = \rho_s g \frac{\pi}{6} d_s^3$

$$= (2400)(9.8) \frac{\pi}{6} (0.001)^3$$

$$= 0.0000123 \text{ N} \quad (2)$$

浮力 $F_b = \gamma_w \nabla_s = \rho_w g \nabla_s = \rho_w g \frac{\pi}{6} d_s^3$

$$= (1000)(9.8) \frac{\pi}{6} (0.001)^3$$

$$= 0.00000513 \text{ N} \quad (3)$$

阻力 $F_D = \frac{1}{2} \rho_w V^2 A C_D$

$$= \frac{1}{2} (1000) V^2 \frac{\pi}{4} (0.001)^2 (0.45)$$

$$= 0.000177 V^2 \text{ N} \quad (4)$$

(2)(3)(4)代入(1)得 $V = 0.2 \text{ m/sec}$

二、【參考題解】

$a = g \tan\theta$

則 $\tan\theta = \frac{3}{9.8} = \frac{c}{10}$

得 $c = 3.06\text{m}$

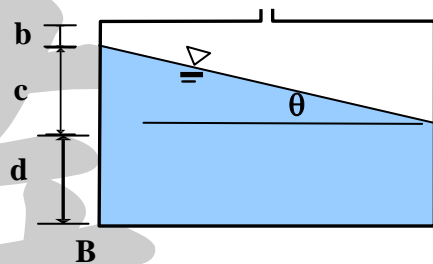
設水未溢出，故移動前油槽內之空氣體積等於移動後油槽內之空氣體積

$$2\left(\frac{\pi}{4}\right)(10)^2 = b \frac{\pi}{4} (10)^2 + \frac{3.06}{2} \frac{\pi}{4} (10)^2$$

得 $b = 0.47\text{m}$

故 $d = 6 - b - c = 6 - 0.47 - 3.06 = 2.47\text{m}$

所以 油槽最大壓力為 $P_B = \rho_{\text{油}} g (c + d) = 0.84(9.8)(3.06 + 2.47) = 45.5\text{KPa}$



三、【參考題解】

平滑圓管的達西-威士巴哈 (Darcy-Weisbach) 式為

$$h_\ell = f \frac{\ell V^2}{D 2g} = \frac{0.316 \ell V^2}{R_e^{1/4} D 2g}$$

管流的水力梯度為 $i = \frac{h_\ell}{\ell} = \frac{0.316}{R_e^{1/4}} \frac{1 V^2}{D 2g} = kD^a Q^b$ (1)

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$
 (2)

$$Re^{1/4} = \left(\frac{\rho V D}{\mu}\right)^{1/4} = \left(\frac{1}{\nu}\right)^{1/4} V^{1/4} D^{1/4} = \left(\frac{1}{\nu}\right)^{1/4} \left(\frac{4Q}{\pi D^2}\right)^{1/4} D^{1/4}$$
 (3)

(2)(3)代入(1)得

$$i = \frac{h_\ell}{\ell} = 0.316 \left(\frac{1}{\nu}\right)^{-1/4} \left(\frac{4Q}{\pi D^2}\right)^{-1/4} D^{-1/4} D^{-1} \frac{1}{2g} \frac{16Q^2}{\pi^2 D^4} = kD^a Q^b$$

即 $0.316 \left(\frac{1}{\nu}\right)^{-1/4} 4^{-1/4} Q^{-1/4} \pi^{1/4} D^{1/2} (D^{-1/4}) D^{-1} \frac{1}{2g} (16) Q^2 D^{-4} \frac{1}{\pi^2} = kD^a Q^b$

各因次合併 $K D^{-19/4} Q^{7/4} = kD^a Q^b$

得係數 $a = \frac{-19}{4}$, $b = \frac{7}{4}$ K 為常數係數

四、【參考題解】

由 N-S 方程式及 B.C ($y=0, u=0, y=d, u=0$)

得速度分布方程式 $u(y) = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 - \frac{d}{2\mu} \frac{\partial p}{\partial x} y$

流量 $Q = \int_0^d u(y)(dy \cdot 1) = \frac{1}{12\mu} \left(\frac{-\partial p}{\partial x}\right) d^3$

平均流速 $\bar{V} = \frac{Q}{d \cdot 1} = \frac{1}{12\mu} \left(\frac{-\partial p}{\partial x}\right) d^2$

當 $y = \frac{d}{2}$ 時, $\frac{du}{dy} = 0$, 所以 $U_0 \frac{\pi}{d} \cos \frac{\pi}{d} y = 0$, 故 $y = \frac{d}{2}$ 時有最大流速 u_{\max}

$$u_{\max} = U_0 = \frac{1}{8\mu} \left(\frac{-\partial p}{\partial x}\right) d^2$$

$$\frac{\bar{V}}{u_{\max}} = \frac{\bar{V}}{U_0} = \frac{2}{3}$$

故 $\bar{V} = \frac{2}{3} U_0 = \frac{2}{3} (0.08) = 0.053 m/s$

$$R_e = \frac{\bar{V} d}{\nu} = \frac{(0.053)(0.04)}{1 \times 10^{-5}} = 212$$